Mech 221: Computer Lab 6

Hand in the solutions to the three questions in the lab at the *end* of the lab.

Point of Clarification: When referring to frequency in this lab, we mean angular frequency.

Question 1: Eigenvalues

Recall the simple harmonic oscillator with damping:

$$m\ddot{x} + \beta\dot{x} + kx = 0 \tag{1}$$

As you have seen before, when the damping β is zero, the solution to the oscillator is given by

$$x(t) = R\cos(\omega_0 t + \phi)$$

where $\omega_0 = \sqrt{k/m}$, is the "natural frequency" of the oscillator. When damping is present, then we have a solution of the form

$$x(t) = Re^{at}\cos(bt + \phi)$$

where b is the natural frequency of oscillations for the damped system, and $a < 0$ is the rate of decay.

- Consider an undamped oscillator, with mass $m = 1$, and spring constant $k = 6$. What is the natural frequency for this system?
- Remember that when the oscillator equation is written as a first order equation it is written as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -k/m & -\beta/m \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2}$$

  Use the `eig` command to find the eigenvalues of the coefficient matrix for the first order system described in the first part of the question. Are they what we expect?
- Now assume that we have the same oscillator with damping, $\beta = 2$. What happens to the eigenvalues with respect to the real and imaginary components? What is the frequency for this oscillator? What is the rate of decay?

- Suppose we increase the damping to $\beta = 4.5$. Will this new level of damping allow oscillations to occur? How can you tell? What about if $\beta = 6$?

- *List the eigenvalues of the first order system above for* $\beta = 0, 2, 4.5$ *and 6 calculated above. For the under-damped cases, list the frequencies and decay rates of the oscillations.*

Note: You can check your answers above by looking at the auxiliary equation for (1). The roots of the auxiliary equation should correspond to the eigenvalues you find.

Question 2: Coupled Oscillators

Remember that the coupled oscillators will satisfy the following equations.

$$\begin{aligned} m_1\ddot{x} &= -k_1 x - K(x-y) - \beta_1\dot{x} \\ m_2\ddot{y} &= K(x-y) - k_2 y - \beta_2\dot{y} \end{aligned} \tag{3}$$

You worked out how to rewrite these equations as a first order system in pre-lab question #2 with a RHS described by a matrix $\mathbf{A}$. It will be helpful for this question and the next to write a .m file that creates the matrix $\mathbf{A}$ for given parameters.

- Take $m_1 = 1$, $m_2 = 1$, $k_1 = 1$, $k_2 = 2$, $K = 3$, $\beta_1 = 0$, and $\beta_2 = 0$ (no damping). What are the natural frequencies of this system? You will be able to determine these from the eigenvalues of $\mathbf{A}$ with the parameters above, using the `eig` command as shown in the pre-lab.

- Repeat the eigenvalue computation above but with $\beta_1 = 0.1$ and $\beta_2 = 0.2$. What are the frequencies and corresponding decay rates in this case?

- Repeat the computation with $\beta_1 = 0$ and $\beta_2 = 100$. Write down the eigenvalues of $\mathbf{A}$ in this case. Discuss the results and explain why they are reasonable physically.

- *Hand in the frequencies in the first, undamped case above, the frequencies and decay rates in the second, under-damped case and the results and discussion of the third case above.*

Question 3: Mystery Coupled Oscillators

For this question you will be given a mystery set of two coupled oscillators. That is, you will be given a set of values of the parameters in the system (3). You will also be given a set of initial conditions.

- Find the frequencies and decay rates of the system you have been given, using eigen-analysis as in the question above.

- Write a .m file function that you can use in `ode45` to solve the first order vector equation that these equations can be transformed to. Solve the problem with the given initial conditions with `ode45` in the interval $0 \le t \le 10$.

- *Hand in the frequencies and decay rates, and the plot of the solution to the initial value problem you computed with* `ode45`.